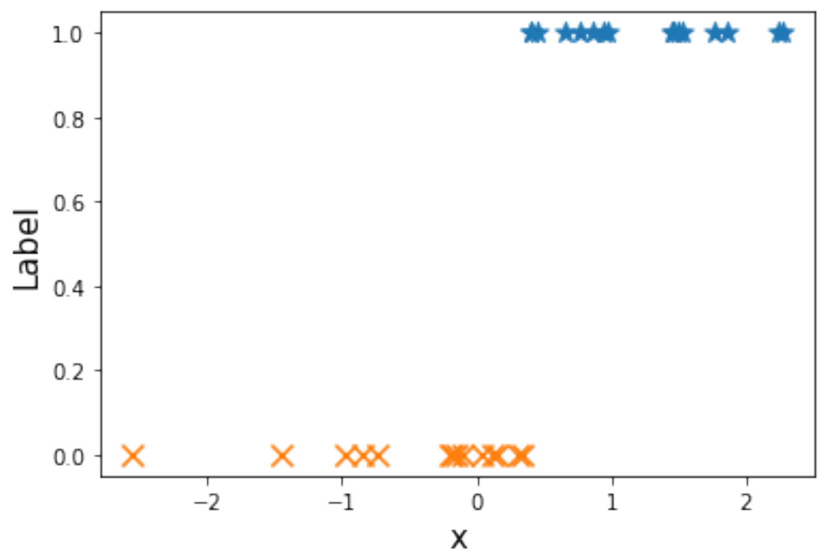
In [4]:
```
import numpy as np
import matplotlib.pyplot as plt
```

Getting the simulated dataset

In [5]:
```
np.random.seed(0)
l = 30
xs = np.random.randn(l)
ys = np.ones(l)
ys[xs < 0.4] = 0
```

In [6]:
```
plt.scatter(xs[ys == 1],ys[ys == 1],marker = '*',s = 100)
plt.scatter(xs[ys == 0],ys[ys == 0],marker = 'x',s = 100)
plt.xlabel('x',size = 15)
plt.ylabel('Label',size = 15)
plt.show()
```



Assuming g(x) represents the sigmoid function

$$g(x) = \frac{1}{1 + e^{-x}}$$

A general form of logistic regression fits model of the form

$$h(x) = g(X\beta) = \frac{1}{1 + e^{-X\beta}}$$

Hence again, like linear regression, we will create a X matrix from the data locations x to fit the logistic Regression model.

Exercise

Since

$$\hat{y} = p(y = 1) = h(x) = \frac{1}{1 + e^{-X\beta}}$$

can we still minimize

$$\min_{\beta} \sum_{i=1}^{n} (y - \hat{y})^2$$

to find the best set of β ?

Optimization problem we have to solve

When a random variable can only assume two possible outcomes, it is a Bernoulli Random Variable. Hence, in the data above, since for every x, y can only be 0 or 1, so x is a Bernoulli Random Variable.

In, order to find the best value of coefficients β, we will write the likelihood of observing the data and then find the value of these β coefficients that will maximize this likelihood.

The likelihood:

$$L = \Pi_{i|y_i=1} h(x_i) \Pi_{i|y_i=0} (1 - h(x_i)) \tag{1}$$

By maximizing L, $h(x_i)$ will be forced to be close to 1 when $y_i = 1$ and $h(x_i)$ will be forced to be close to 0 when $y_i = 0$. Hence if

$$h(x_i) = \frac{1}{1 + e^{-X\beta}}$$

An algorithm maximizing L, will try to find coefficients β that makes $h(x_i) \rightarrow 1$ when $y_i = 1$, and $h(x_i) \rightarrow 0$ when $y_i = 0$

Changing likelihood to negative log likelihood

Since maximizing the product terms in (1) is challenging, we convert (1) to an equivalent problem form by taking a log (converts products to sum of terms) and multiplying a negative 1. So essentially, we have following equivalent problems

$$\max_{\beta} L \equiv \min_{\beta} -log(L) \equiv \min_{\beta} LL \quad \text{where } LL = -log(L)$$

$$
\begin{aligned}
LL &= -log(L) \\
&= -\sum_{i|y_i=1} log(h(x_i)) - \sum_{i|y_i=0} log(1 - h(x_i)) \\
&= -\sum_{i} \left[y_i log(h(x_i)) + (1 - y_i) log(1 - h(x_i)) \right]
\end{aligned}
$$

Hence we have to solve the optimization problem:

$$
\min_{\beta} LL = \boxed{\min_{\beta} - \sum_{i} \left[y_i log(h(x_i)) + (1 - y_i) log(1 - h(x_i)) \right]} \tag{2}
$$

where

$$
h(x) = \frac{1}{1 + e^{-X\beta}}
$$

Remember

1. The above loss function is convex
2. Dont have a closed form solution like linear Regression
3. Hence, we use iterative algorithms to find the solution

```
In [ ]:
```